An asymptotical separability criterion for bipartite density operators

Romàn R. Zapatrin*

February 9, 2008

Abstract

For a given density matrix ρ of a bipartite quantum system an asymptotical separability criterion is suggested. Using the continuous ensemble method, a sequence of separable density matrices is built which converges to ρ if and only if ρ is separable. The convergence speed is evaluated and for any given tolerance parameter κ an iterative procedure is suggested which decides in finite number of steps if there exists a separable density matrix ρ_κ which differs from the matrix ρ by at most κ.

1 Introduction

The notion of entanglement plays a central rôle in quantum communication and quantum computation. Usually the efforts are focused on quantifying entanglement itself, that is, describing the *impossibility* to prepare a state by means of LOCC (local operations and classical communications). One may, although, try to quantify *separability* rather than entanglement, this was shown to be a tool for geometrical classification of mixed multipartite entangled states [7]. I dwell on the case of bipartite quantum systems. A state of such system is called separable if it can be prepared by LOCC. In terms of density matrices that means that $\boldsymbol{\rho}$, its density matrix, can be represented as a mixture of pure product states. I suggest to replace finite sums of projectors by continuous distributions on the set of unit vectors.

Recently continuous ensemble method was introduced [8] which was applied to explore the separability of finite-dimensional quantum systems

*Friedmann Lab. for Theoretical Physics, SPb EF University, Griboyedova 30–32, 191023, St.Petersburg, Russia; e-mail: zapatrin@rusmuseum.ru

[9, 10]. First, a geometrical characterization of robustly separable mixed states was provided in [9], then the separability problem was reduced to a finite number of numerical equations [10]. This paper suggests an iterative procedure which, for any given tolerance parameter, decides if the state is separable or not for finite number of steps.

This is done in the following way. In bipartite case the density operators are represented by distribution on the Cartesian product of unit spheres in subsystems' spaces. Given a density operator $\boldsymbol{\rho}$, we consider it as an element of the space $\mathfrak{L}$ of all self adjoint operators in the product space $\mathfrak{B} = \mathcal{H} \otimes \mathcal{H}$. The density operator $\boldsymbol{\rho}$ is called ROBUSTLY SEPARABLE if it has a neighborhood of separable operators [6]. The robust separability of $\boldsymbol{\rho}$ is shown [10] to be equivalent to the solvability of the following vector equation in $\mathfrak{L}$:

$$\nabla \mathbf{K} \;=\; \boldsymbol{\rho}$$

with respect to the trace functional $\mathbf{K}$ on $\mathfrak{L}$ introduced in the first paper [9] of this series. When we fix a product basis in $\mathfrak{B}$, this equation becomes a system of n^4 transcendent equations with respect to n^4 variables [10].

In this paper an asymptotic procedure based on iterated function method is suggested which, in case if $\boldsymbol{\rho}$ is separable, approximates $\boldsymbol{\rho}$ by a sequence of separable density operators, otherwise this sequence diverges. The operator $\mathbf{F}$ is introduced

$$\mathbf{F}(X) \;=\; X + \lambda \left(\boldsymbol{\rho} - \iint e^{\langle \phi\phi' | X | \phi\phi' \rangle} \left| \phi\phi' \right\rangle \!\! \left\langle \phi\phi' \right| \, d\mathbf{S}_n \, d\mathbf{S}'_n \right)$$

where the integration is taken over the torus—the Cartesian product of unit spheres in $\mathcal{H}, \mathcal{H}'$. Then the sequence of operators $X_0, X_1, \ldots, X_m, \ldots$ is built

$$\begin{cases} X_0 = 0 \\ X_1 = \mathbf{F}(X_0) \\ \ldots \\ X_m = \mathbf{F}(X_{m-1}) \\ \ldots \end{cases}$$

which converges if $\boldsymbol{\rho}$ is robustly separable. The constant λ and the speed of this convergence are (roughly) evaluated, they depend on the dimension n of the state space and the tolerance κ we choose.

2 Continuous optimal ensembles

In this section I briefly summarize the basics of continuous ensembles approach to quantifying quantum separability. In general context related to coherent states, continuous ensembles were considered yet by Glauber [1] and Sudarshan [5]. Generalizing the fact that any convex combination of density operators is again a density operator, we represent density operators as probability distributions on the unit sphere[1] in the state space $\mathcal{H}$ of the system (these ideas were also considered in [3]). In particular, the well-known spectral decomposition of an operator is a special case of such representation, this will be addressed below. However the main feature of the suggested approach is that the distribution is taken over *non-orthogonal* states.

Begin with the case of a single quantum system. Consider a probability distribution on $\mathbb{C}B_n$ whose density is a function $\mu(\phi)$, where $\phi \in \mathbb{C}B_n$ ranges over all unit vectors in $\mathcal{H}$. The density operator of this continuous ensemble is:

$$\rho \;=\; \int\limits_{\phi\in\mathbb{C}B_n} \mu(\phi)\; |\phi\rangle\langle\phi|\;\; d\mathbf{S}_n \tag{1}$$

where $|\phi\rangle\langle\phi|$ is the projector onto the vector $\langle\phi|$ and $d\mathbf{S}_n$ is the normalized measure on $\mathbb{C}B_n$. Effectively the operator integral ρ (1) can be calculated by its matrix elements. In any fixed basis $\{|\mathbf{e}_i\rangle\}$ in $\mathcal{H}$, the matrix elements $\rho_{ij} = \langle\mathbf{e}_i|\,\rho\,|\mathbf{e}_j\rangle$ are the following *numerical* integrals:

$$\rho_{ij} \;=\; \langle\mathbf{e}_i|\,\rho\,|\mathbf{e}_j\rangle \;=\; \int\limits_{\phi\in\mathbb{C}B_n} \mu(\phi)\; \langle\mathbf{e}_i\,|\,\phi\rangle\,\langle\phi\,|\,\mathbf{e}_j\rangle\;\; d\mathbf{S}_n$$

Smeared spectral decomposition. Since we are interested in robustly separable states, let us restrict ourselves to full-range density operators. The usual spectral decomposition $\rho = \sum p_k\,|\mathbf{e}_k\rangle\langle\mathbf{e}_k|$ can be treated as an atomic measure on $\mathbb{C}B_n$ whose density is the appropriate combinations of delta functions:

[1] Pure states form a projective space rather than the unit sphere in $\mathcal{H}$. On the other hand, one may integrate over any probabilistic space. For technical reasons I prefer to represent ensembles of pure states by measures on unit vectors in $\mathcal{H}$. I use the Umegaki measure on $\mathbb{C}B_n$— the uniform measure with respect to the action of $U(n)$ normalized so that $\int_{\mathbb{C}B_n} d\mathbf{S}_n = 1$. Similarly, for bipartite case the integration will be carried out over the Cartesian product of unit spheres in appropriate state spaces.

$$\mu_{\mathrm{spec}}(\phi) = \sum p_k \, \delta(\phi - \mathbf{e}_k)$$

We introduce a 'smeared' version of the spectral decomposition. Let K be an integer number, then the density matrix ρ is represented as a continuous ensemble:

$$\rho \;=\; \sum p_k \, |\mathbf{e}_k\rangle\langle\mathbf{e}_k| \;=\; \int\limits_{\phi\in\mathbb{C}B_n} \mu(\phi)\; |\phi\rangle\langle\phi|\;\, d\mathbf{S}_n$$

The density μ was calculated in [10], it equals

$$\mu(\phi) \;=\; \frac{((K+1)n)!}{K\cdot(Kn)!\,n!}\cdot\sum_k \left(p_k \,-\, \frac{1}{(K+1)n}\right) |\langle \mathbf{e}_k \mid \phi\rangle|^{2Kn} \qquad (2)$$

Furthermore, the distribution (2) tends to the spectral decomposition of ρ as K tends to infinity. Denote by p_0 the smallest eigenvalue of the density matrix ρ (recall that $p_0 > 0$ as we restrict ourselves to full-range density matrices). Then the density μ in (2) is positive for any such K that $(K+1)n \geq 1/p_0$. We may take

$$K \;=\; \frac{1}{n\,p_0} \qquad (3)$$

Optimal entropy ensembles. It was shown in [9, 10] that, given a robustly separable density matrix $\boldsymbol{\rho}$ in the product space $\mathfrak{B} = \mathcal{H}\otimes\mathcal{H}'$, there exists a self-adjoint operator X in $\mathfrak{B}$ such that $\boldsymbol{\rho}$ can be represented as the following operator integral:

$$\boldsymbol{\rho} \;=\; \int\limits_{\phi\phi'\in\mathfrak{T}} \int e^{\langle\phi\phi'|X|\phi\phi'\rangle}\, \big|\phi\phi'\big\rangle\big\langle\phi\phi'\big|\; d\mathbf{S}_n\, d\mathbf{S}'_n \qquad (4)$$

Fix a product basis $\{\langle \mathbf{e}_i\mathbf{e}'_{i'}|\}$. Then the condition (4) takes the form

$$\big\langle \mathbf{e}_i\mathbf{e}'_{i'}\big|\,\boldsymbol{\rho}\,\big|\mathbf{e}_j\mathbf{e}'_{j'}\big\rangle \;=\; \int\limits_{\phi\phi'\in\mathfrak{T}} \int e^{\langle\phi\phi'|X|\phi\phi'\rangle}\, \big\langle \mathbf{e}_i\mathbf{e}'_{i'}\;\big|\;\phi\phi'\big\rangle\big\langle\phi\phi'\;\big|\;\mathbf{e}_j\mathbf{e}'_{j'}\big\rangle\; d\mathbf{S}_n\, d\mathbf{S}'_n \qquad (5)$$

and the separability problem reduces to finding the operator X from this system of equations. The essence of the separability problem is the *existence* of a solution of (5). In this paper I suggest an asymptotic procedure which gives the answer up to an arbitrary (but finite) precision.

3 Contraction mappings and the iterated function method

A CONTRACTION MAPPING is any mapping $\mathbf{F} : \mathcal{D} \to \mathcal{D}$ in a metric space $\mathcal{D}$ such that the distance d between any two points before the mapping is greater than the distance between them after the mapping:

$$\exists\, C < 1 \quad \forall\, x, y \in \mathcal{D} \qquad d(\mathbf{F}(x), \mathbf{F}(y)) \;\leq\; C \cdot d(x, y) \tag{6}$$

The most important property of a contraction mapping is that there is exactly one point which is invariant under the mapping. The second crucial feature is that under sufficiently long iteration every point eventually contracts to this point. Using the definition (6), the speed of convergence can be evaluated from the value of the contraction parameter C and the diameter D of the space $\mathcal{D}$. After N steps the distance between the fixed point of $\mathbf{F}$ and its N-th approximation will not exceed $C^N D$, therefore the given accuracy κ will be achieved in at most

$$N \;=\; \frac{\ln \kappa - \ln D}{\ln C} \tag{7}$$

steps. A standard way to solve the equation $f(x) = a$ using the iterated function method is to define the function $F(x)$:

$$F(x) \;:=\; x + \lambda(a - f(x))$$

and choose a domain $\mathcal{D}$ for x such that f acts within $\mathcal{D}$ and a parameter λ in order to make f contracting on $\mathcal{D}$. If the derivative f' is bounded on $\mathcal{D}$, then the contraction condition can always be satisfied by choosing sufficiently small λ, namely, take any

$$\lambda \;<\; \frac{1}{\max |f'|}$$

In this case the contraction parameter in (6) is $C = \lambda / \max |f'|$.

Now let us apply this method to the equation (5). Introduce the mapping $F : \mathfrak{L} \to \mathfrak{L}$:

$$\mathbf{F}(X) \;=\; X + \lambda \left(\boldsymbol{\rho} \;-\; \int\!\!\int_{\phi\phi' \in \mathfrak{T}} e^{\langle \phi\phi' | X | \phi\phi' \rangle} \; \big|\phi\phi'\big\rangle\big\langle\phi\phi'\big| \; d\mathbf{S}_n \, d\mathbf{S}'_n \right) \tag{8}$$

with the domain $\mathcal{D} = \{X : \|X\| \leq A\}$ for a sufficiently large A, see section 4 for evaluations of the parameters in (8). If such X exists, then

- we can always choose λ which makes $\mathbf{F}$ contraction mapping (this is because $e^{\langle\phi\phi'|X|\phi\phi'\rangle}$ is bounded on $\mathcal{D}$)
- whatever be $\boldsymbol{\rho}$, the zero operator 0 is always in $\mathcal{D}$

This opens us the way to solve the separability problem. Start the iteration procedure with the zero operator, then, if the solution exists, the sequence will converge, otherwise not.

4 Evaluating the parameters

In order to evaluate the parameters for the applicability of the iterated function method, we need to specify

- The domain $\mathcal{D}$—to make the function $\mathbf{F}$ act within $\mathcal{D}$
- The value of the parameter λ—to make the mapping $\mathbf{F}$ contracting

Assume that the density operator $\boldsymbol{\rho}$ is separable, then there exists a finite set of 1-dimensional product projectors $\{\langle \mathbf{f}_\alpha \mathbf{f'}_\alpha|\}$ such that $\boldsymbol{\rho}$ is a convex combination

$$\boldsymbol{\rho} \;=\; \sum_\alpha c_\alpha \left|\mathbf{f}_\alpha \mathbf{f'}_\alpha\right\rangle\!\left\langle \mathbf{f}_\alpha \mathbf{f'}_\alpha\right|$$

The (real) dimension of the state of all self-adjoint operators in the product space $\mathfrak{B}$ is n^4. According to Karathéodori theorem, the projectors $|\mathbf{f}_\alpha \mathbf{f'}_\alpha\rangle\langle \mathbf{f}_\alpha \mathbf{f'}_\alpha|$ can be chosen so that their number in the above sum will not exceed n^8.

Now recall that we have a tolerance parameter κ. What we need, is to assume that each product projector $|\mathbf{f}_\alpha \mathbf{f'}_\alpha\rangle\langle \mathbf{f}_\alpha \mathbf{f'}_\alpha|$ can be replaced by the mixture

$$\boldsymbol{\rho}_\alpha \;=\; \left(\left(1-\frac{\kappa}{n^8}\right)\cdot|\mathbf{f}_\alpha\rangle\langle\mathbf{f}_\alpha| \;+\; \frac{\kappa}{n^8}\,\mathbb{I}\right) \otimes \left(\left(1-\frac{\kappa}{n^8}\right)\cdot\left|\mathbf{f'}_\alpha\right\rangle\!\left\langle\mathbf{f'}_\alpha\right| \;+\; \frac{\kappa}{n^8}\,\mathbb{I}\right) \tag{9}$$

Smeared spectral decomposition for the summands. Each summand in (9) is a product of density matrices in single-party spaces. Represent it as a continuous mixture

$$\boldsymbol{\rho}_\alpha \;=\; \left(\int\limits_{\mathbb{C}B_n} \mu_\alpha(\phi)\;|\phi\rangle\langle\phi|\;d\mathbf{S}_n\right)\otimes\left(\int\limits_{\mathbb{C}B_n} \mu_\alpha(\phi')\;|\phi'\rangle\langle\phi'|\;d\mathbf{S}'_n\right) \qquad (10)$$

Let us first evaluate the range $\mu_0 < \mu < \mu_1$ of the density $\mu(\phi)$ for the single particle case. First note that the values of the uniform function $f = \sum c_k r_k^{2m}$ on the sphere $\sum r_k^2 = 1$ ranges between

$$\frac{c_0}{n^{m-1}} \le f(r_1,\ldots,r_n) \le c_1$$

where c_0 and c_1 are the least and the greatest coefficients, respectively. Then it follows from (2) that

$$\frac{((K+1)n)!}{K\cdot(Kn)!\,n!}\cdot\frac{1}{n^{Kn-1}}\left(p_0 - \frac{1}{(K+1)n}\right) \le \mu \le \frac{((K+1)n)!}{K\cdot(Kn)!\,n!}\cdot\left(p_1 - \frac{1}{(K+1)n}\right)$$

Denote

$$C_K \;=\; \frac{((K+1)n)!}{K\cdot(Kn)!\,n!} \qquad (11)$$

For evaluation purposes replace $p_1 - \frac{1}{(K+1)n}$ by 1. Substituting $K = 1/np_0$ from (3) and calculating $p_0 - \frac{1}{(K+1)n} = \frac{1}{n}\cdot\frac{n^2p_0^2}{1+np_0} > \frac{1}{n}\cdot\frac{n^2p_0^2}{2}$ we obtain

$$C_K\;\cdot\;\frac{1}{n^{Kn}}\;\cdot\;\frac{n^2p_0^2}{2} \le \mu \le\; C_K \qquad (12)$$

Recall that we consider our density operator up to a tolerance parameter κ, hence we may assume from (9) that the smallest eigenvalue p_0 in (2) to be at least κ/n^8, therefore K can be evaluated from (3) as

$$K \;=\; \frac{1}{np_0} \;=\; \frac{n^7}{\kappa} \qquad (13)$$

Continue the evaluation (12) with $p_0 = \kappa/n^8$:

$$C_K\;\cdot\;\frac{1}{n^{(n^8/\kappa)}}\;\cdot\;\frac{\kappa^2}{2n^{14}} \le \mu \le\; C_K$$

Then denote

$$C_A \;=\; \frac{2n^{14}n^{(n^8/\kappa)}}{\kappa^2} \tag{14}$$

Taking into account that $C_K, C_A >> 1$, make the evaluation of μ rough but symmetric:

$$\frac{1}{C_K \cdot C_A} \leq \mu \leq C_K \cdot C_A$$

then the magnitude of the density representing of each product $\boldsymbol{\rho}_\alpha$ in (10) ranges within

$$\left(\frac{1}{C_K C_A}\right)^2 \leq \mu_\alpha(\phi) \cdot \mu_\alpha(\phi') \leq C_K^2\, C_A^2$$

Since the initial density matrix $\boldsymbol{\rho}$ is a convex combinations of the matrices $\boldsymbol{\rho}_\alpha$, the matrix $\boldsymbol{\rho}$ itself can be represented as a continuous ensemble whose density ranges within the same bounds (15). That means that the values of the operator X satisfy the following relations

$$\left(\frac{1}{C_K C_A}\right)^2 \;\leq\; e^{\langle\phi\phi'|X|\phi\phi'\rangle} \;\leq\; C_K^2\, C_A^2 \tag{15}$$

This gives us the domain for the function $\mathbf{F}$ we are going to iterate:

$$\|X\| \;\leq\; 2\ln\left(C_K C_A\right) \tag{16}$$

where the expression for C_K and C_A are obtained above in (11) and (14), respectively, and the norm is L_1.

Evaluating the derivative. In order to learn the value of the parameter λ in (8) the derivative of $\mathbf{F}$ in any arbitrary direction must be evaluated. Let Y be a self-adjoint operator in $\mathfrak{B}$, consider the operator $\mathbf{F}(X+tY)$ and calculate its derivative in the direction Y at $t=0$:

$$D \;:=\; \frac{d}{dt}\mathbf{F}(X+tY)\bigg|_{t=0} \;=\; Y - \lambda \int\limits_{\phi\phi'\in\mathfrak{T}} e^{\langle\phi\phi'|X|\phi\phi'\rangle} \left\langle\phi\phi'\right| Y \left|\phi\phi'\right\rangle \left|\phi\phi'\right\rangle\!\left\langle\phi\phi'\right| \, d\mathbf{S}_n \, d\mathbf{S}'_n \tag{17}$$

Note that, according to (15), the values of $e^{\langle\phi\phi'|X|\phi\phi'\rangle}$ are bounded by $(C_K C_A)^2$, therefore, in order to evaluate the derivative, we take the maximal value, so the value of D is bounded

$$\|D\| \;\leq\; \left\| Y - \lambda\,(C_K C_A)^2 \int\!\!\int\limits_{\phi\phi'\in\mathfrak{T}} \left\langle \phi\phi' \right| Y \left| \phi\phi' \right\rangle \, \left| \phi\phi' \right\rangle\!\left\langle \phi\phi' \right| \, d\mathbf{S}_n \, d\mathbf{S}'_n \right\|$$

In order to evaluate the integral I use the formula obtained in [8] for the single particle case. For any self-adjoint operator A in $\mathcal{H}$

$$\int\limits_{\phi\in\mathbb{C}B_n} \langle\phi|\, A\,|\phi\rangle\, |\phi\rangle\langle\phi| \;\, d\mathbf{S}_n \;=\; \frac{A + \mathbb{I}\cdot \operatorname{Tr} A}{n(n+1)}$$

Recall that the L_1 norm is used, this makes it possible to evaluate the traces of Y as they are bounded by the norm of Y, then

$$\left\| A \;-\; \int\limits_{\phi\in\mathbb{C}B_n} \langle\phi|\, A\,|\phi\rangle\, |\phi\rangle\langle\phi| \;\, d\mathbf{S}_n \right\| \;\leq\; \|A\| \cdot \left(1 - \frac{1}{n(n+1)}\right) \tag{18}$$

Since the integral (17) is a linear function of the operator Y,

$$\int\!\!\int\limits_{\phi\phi'\in\mathfrak{T}} \left\langle \phi\phi' \right| Y \left| \phi\phi' \right\rangle \, \left| \phi\phi' \right\rangle\!\left\langle \phi\phi' \right| \, d\mathbf{S}_n \, d\mathbf{S}'_n \;=\; \frac{Y + \mathbb{I}\otimes \operatorname{Tr_I} Y + \operatorname{Tr_{II}} Y \otimes \mathbb{I} + \operatorname{Tr} Y \cdot \mathbb{I}\otimes\mathbb{I}}{n^2\,(n+1)^2}$$

where $\operatorname{Tr_I}, \operatorname{Tr_{II}}$ stand for partial traces. So, taking into account (18)

$$\|D\| \;\leq\; \left\| Y - \lambda\,(C_K C_A)^2\, \frac{Y + \mathbb{I}\otimes \operatorname{Tr_I} Y + \operatorname{Tr_{II}} Y \otimes \mathbb{I} + \operatorname{Tr} Y \cdot \mathbb{I}\otimes\mathbb{I}}{n^2\,(n+1)^2} \right\|$$

When we set λ to be

$$\lambda \;=\; \frac{1}{(C_K C_A)^2} \tag{19}$$

we have the following evaluation of contraction constant $\|D\| \leq C\|Y\|$ in (6):

$$C \;=\; \left(1 - \frac{1}{n(n+1)}\right)^2 \tag{20}$$

Evaluating the speed of convergence. According to (16) we have the following evaluation of the diameter of the domain X

$$D \;\leq\; 4\ln(C_A C_K)$$

Then, fixing a tolerance parameter κ and using the calculated contraction constant (20) we can apply (7) to find the sufficient number of iterations

$$N \;=\; \frac{\ln\kappa - \ln\bigl(4\ln(C_A C_K)\bigr)}{2\ln\left(1-\frac{1}{n(n+1)}\right)}$$

and approximating the logarithm in the denominator we get

$$N \;=\; 2n(n+1)\left(\ln\bigl(4\ln(C_A C_K)\bigr) + \ln\frac{1}{\kappa}\right) \tag{21}$$

5 Concluding remarks

Given a density matrix $\boldsymbol{\rho}$ in $\mathfrak{B}$, a question arises if it is separable or not. When the dimension of at least one of spaces $\mathcal{H}, \mathcal{H}'$ is 2, this question was given an effective and exact answer—the positive partial transpose (PPT) criterion due to Peres-Horodecki was suggested [4, 2]. The criterion states that $\boldsymbol{\rho}$ is separable if and only if its partial transpose $\boldsymbol{\rho}^{T_{II}}$ remains non-negative matrix. In higher dimensions PPT is only a necessary condition for a state to be factorizable as there exist entangled density matrices whose partial transpose if positive.

In this paper the dimension of $\mathcal{H}, \mathcal{H}'$ may take any finite value n. The separabilty problem is resolved with any given in advance finite precision. Let us summarize the procedure.

- Given a density operator $\boldsymbol{\rho}$ in the product space $\mathfrak{B} = \mathcal{H}\otimes\mathcal{H}' \simeq \mathbb{C}^{n^2}$, calculate its minimal eigenvalue p_0.
- Set up the tolerance parameter κ and calculate the values of K (13), C_K (11) and C_A (14):

  $$K \;=\; \frac{n^7}{\kappa} \quad ; \qquad C_K \;=\; \frac{((K+1)n)!}{K\cdot(Kn)!\,n!} \quad ; \qquad C_A \;=\; \frac{2n^{14}n^{(n^8/\kappa)}}{\kappa^2}$$

  from which we derive the value of λ (19) and the number of steps N (21):

  $$\lambda \;=\; \frac{1}{(C_K C_A)^2} \quad ; \qquad N \;=\; 2n(n+1)\left(\ln\bigl(4\ln(C_A C_K)\bigr) + \ln\frac{1}{\kappa}\right)$$

- Iterate the function $\mathbf{F}$ (8)

$$X_{k+1} \;=\; \mathbf{F}(X_k) \;=\; X_k + \lambda \left(\boldsymbol{\rho} \;-\; \int\!\!\int\limits_{\phi\phi' \in \mathfrak{T}} e^{\langle \phi\phi' | X_k | \phi\phi' \rangle} \; \left| \phi\phi' \right\rangle\!\left\langle \phi\phi' \right| \, d\mathbf{S}_n \, d\mathbf{S}'_n \right)$$

  starting from $X_0 = 0$. At each step check $||X||_1$ and if it occurs that $||X||_1 > 2 \ln (C_K C_A)$ that signals us that $\boldsymbol{\rho}$ is entangled.

- After N steps we get the approximation of $\boldsymbol{\rho}$ with accuracy κ

I emphasize that this method is not completely suited for practical calculations as the evaluations for the convergence parameters are very rough. This paper just demonstrates the principal possibility of using the continuous ensemble method to tackle the separability problem.

Acknowledgments. I would like to express my gratitude to many people who offered me their attention for a pretty long time while this work was carried out. These are Abhay Ashtekar, Paolo Giorda, Markus Grassl, Radu Ionicioiu, Serguei Krasnikov, Rafael de la Madrid, Ioannis Raptis, Mario Rasetti, Martin Rötteler, Ujjwal Sen, Karl Svozil, Xiaoguang Wang, Paweł Wojcian, Christof Zalka and, alphabetically the last but definitely not the least, Paolo Zanardi.

The financial support for the initial stage of this research was provided by the EU FP5 Project Q-ACTA (in the I.S.I. Foundation, Torino, Italy). Crucial issues related to this research were intensively discussed during the meeting Glafka-2004 'Iconoclastic Approaches to Quantum Gravity' (15–18 June, 2004, Athens, Greece) supported by QUALCO Technologies (special thanks to its organizers—Ioannis Raptis and Orestis Tsakalotos). The financial support on the final stage was provided by the research grant No. 04-06-80215a from RFFI (Russian Basic Research Foundation).

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